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### Semester Two Examination, 2019

### Question/Answer booklet

# MATHEMATICS SPECIALIST

**UNIT 2**

## Section One:

## Calculator-free

Your Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Your Teacher’s Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

| Question | Mark | Max | Question | Mark | Max |
| --- | --- | --- | --- | --- | --- |
| 1 |  | 3 | 6 |  | 8 |
| 2 |  | 6 | 7 |  | 6 |
| 3 |  | 7 | 8 |  | 4 |
| 4 |  | 8 | 9 |  | 3 |
| 5 |  | 7 |  |  |  |

**Structure of this paper**

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| --- | --- | --- | --- | --- | --- |
| Section One:Calculator-free | 9 | 9 | 50 | 52 | 36 |
| Section Two:Calculator-assumed | 13 | 13 | 100 | 94 | 64 |
|  |  |  |  | **Total** | 100 |

**Instructions to candidates**

1. The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2019*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.
5. **Show all your working clearly.**Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula sheet is **not** to be handed in with your Question/Answer booklet.

**See Next Page**

**Section One: Calculator-free (52 Marks)**

This section has **nine (9)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

● Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.

● Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

**Question 1 (3 marks)**

Prove the following statement:

If $a$ and $b$ are each 1 less than a multiple of 3, then $ab$ is 1 more than a multiple of 3.

**Question 2 (6 marks)**

Consider the system of simultaneous linear equations:

$3x-ay=6 -6x+4y=b$

1. Write down the matrix $A$ such that the equation

$A\left[x y \right]=\left[6 b \right]$

is equivalent to the system of equations above. (1 mark)

1. Suppose that $A$ is singular (non-invertible).
2. Determine the value of $a$ (show working). (2 marks)
3. State the possible number(s) of solutions that the system of equations could have with the value of $a$ you just found. (2 marks)
4. State the number of solutions the system has if $a$ has the value found above and $b=11$. (1 mark)

**Question 3 (7 marks)**

1. Write $3coscos 5x +3\sqrt{3}sinsin 5x $ in the form $asinsin (bx+α) $. (3 marks)
2. Hence, solve the equation $3coscos 5x +3\sqrt{3}sinsin 5x =3\sqrt{3}$ for $-\frac{π}{2}\leq x\leq \frac{π}{2}$. (4 marks)

**Question 4 (8 marks)**

A $2×2$ real matrix $A$ can ‘transform’ a complex number if we view the complex number as a column vector. That is, for any complex number $z=a+bi$, the matrix $A$ transforms $z$ to $c+di$ where $\left[c d \right]=A\left[a b \right]$.

Find the matrix $A$ which (according to this rule) will transform any complex number $z$ to:

1. $3z$ (2 marks)
2. $\overline{z}$ (2 marks)
3. $iz$ (2 marks)
4. $i\overline{z}$ (2 marks)

**Question 5 (7 marks)**

Evaluate the following for complex numbers $z=2+5i$ and $w=1-4i$

1. $z-w$ (2 marks)
2. $z(w+\overline{w})$ (2 marks)
3. $\frac{w}{z}$ (3 marks)

**Question 6 (8 marks)**

In this question, a *proper* factor is a factor greater than 1.

Assume that $a$ and $b$ are both integers, and consider the following statement:

If $ab$ has no proper square factors, then neither $a$ nor $b$ has a proper square factor.

1. Prove the statement using the method of proof by contradiction. (3 marks)
2. Write the converse of the statement. (2 marks)
3. State whether the converse is true or false and prove or disprove it accordingly.

 (3 marks)

**Question 7 (6 marks)**

Let $O$ be the origin, let $A$ and $B$ be points such that $OA=OB$, and let $C$ be a point on $\overline{AB}$ such that $\overline{OC}$ bisects $∠AOB$.

Let $a=\vec{OA}$, $b=\vec{OB}$ and $c=\vec{OC}$.

a) Show that $a⋅c=b⋅c$. (3 marks)

b) **Hence**, prove that $\overline{OC}$ is perpendicular to $\overline{AB}$. (3 marks)

**Question 8 (4 marks)**

Prove the following identity.

$\frac{sin 7θ -sin 2θ }{cos 2θ +cos 7θ }=tantan \frac{5θ}{2} $

**Question 9 (3 marks)**

Let $l$ be a line containing a point $P$, and let $Q$ be a point not on $l$. Suppose that $\hat{n}$ is a unit vector perpendicular to the line $l$. Prove that the perpendicular distance from $Q$ to $l$ is $\left|\vec{PQ}⋅\hat{n}\right|$.

**END OF SECTION ONE**

**Additional working space**

Question number:

**Additional working space**

Question number: